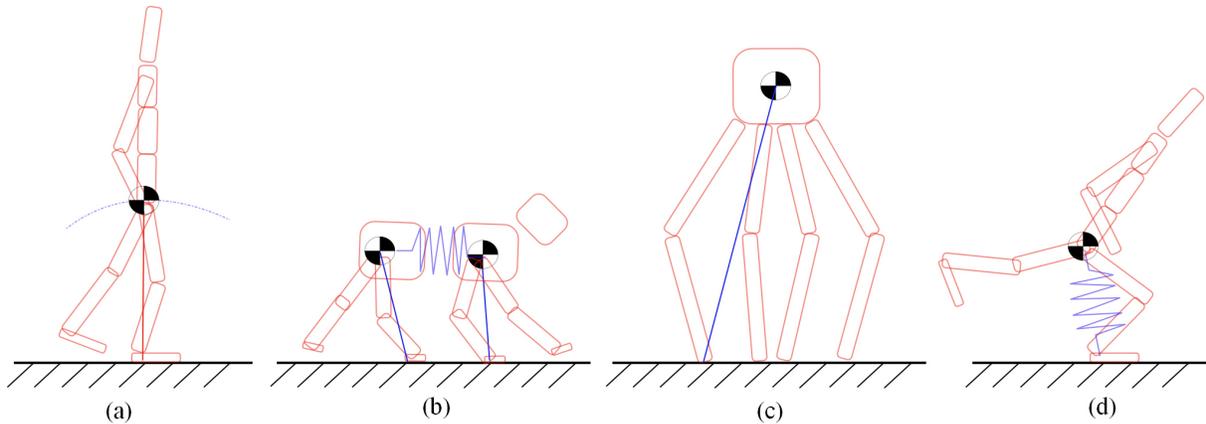


# Character Inverted Pendulum

## Pogo-Sticks, Pole-Vaulting, and Dynamic Stepping

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**Figure 1:** Creative adaptation of the IP model for creating balancing characters solutions. Extending and adapting the inverted pendulum technique to different skeleton character configurations. (a) incompressible leg, (b) combined inverse pendulums (i.e., quadruped), (c) multiple legs and, (d) spring-loaded inverted pendulum (SLIP).

### Abstract

This article gives a practical overview of the popular biomechanically inspired, computationally efficient, algorithmically straightforward inverted pendulum technique for character-based systems. We explain the different flavours of inverted pendulum (e.g., spring-loaded and gravity compensated inverted pendulum), their viability for different situations (e.g., walking, running), simulation results, and practical step-by-step implementation details. We also discuss how the inverted pendulum model can be used for biped and multi-leg characters (e.g., humans and dogs) and any necessary engineering solutions that might be necessary to make the implementation a practical usable solution for real-time environments. While a basic introduction introduces the mathematics and principles behind the inverted pendulum they can brush over or neglect to mention numerical approximations and corrective engineering solutions necessary to make the inverted pendulum a usable tool for character based control (e.g., upright balanced walking). The inverted pendulum is a self-adapting low-dimensional controller that provides intelligent foot placement information for balancing and upright locomotion.

**CR Categories:** I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Animation I.6.8 [Computer Graphics]: Animation—Types of Simulation—Animation

**Keywords:** Procedural, Inverted Pendulum, Character, Walking, Practical, Balancing, Biped, Responsive, Dynamic, Pogo-Sticks, Pole-Vaulting, Stepping

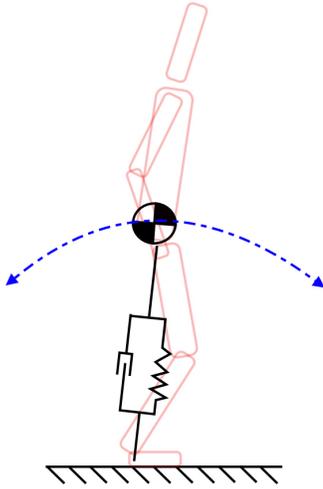
### 1 Introduction

Creating physically accurate life-like characters that are dynamic, interactive and customizable is complex and challenging. This paper focuses on upright balancing motions (e.g., standing and walking). A popular method that has been used greatly to produce robust, reliable, and controllable balancing information is the inverted pendulum. The inverted pendulum has a number of flavours (i.e., variations) that we discuss. Understanding the actual 'concept' of how the inverted pendulum works and it can compensate for errors is the most crucial factor for implementing a robust and viable solution. Since a lot of the theory is based upon ideal situations (e.g., perfectly flat terrain) and do not incorporate dynamic unpredictable situations from changing terrain (e.g., walking across a bridge) or numerical errors from floating point approximations. We discuss the fundamentals, mathematics, logic, and what we need to add to the inverted pendulum model to make it practical character-based solution; such as prediction, interpolation, and engineering adjustments necessary to compensate for numerical errors. A good ex-

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ample of a situation that demonstrates the potential of the dynamic properties of a physics-based character is that of situations with unforeseen force disturbances occur, such as pushes and trips in interactive environments like games. Additionally, walking on a swing bridge where the terrain can be changing and moving and the character constantly has to adapt and move to compensate from falling.



**Figure 2: Illustrating Inverted Pendulum (IP) Biped Principle** - Calculating stance and locomotion stepping information for a biped model using the computationally fast and simple spring-damper inverted pendulum model.

**What makes the IP so special?** Essentially, the inverted pendulum is attractive because it is very straightforward to understand and computationally efficient (i.e., run at real-time frame-rates). Furthermore, it provides a physically-accurate low-dimensional model of a character's balancing stepping movement that can be used to correct or generate existing animations.

### 1.1 Motivation

The motivation behind this work is dominated by the goal of producing more flexible, dynamic, physically-accurate, life-like characters that go beyond traditional hard-coded key-framed animation libraries towards procedurally intelligent motions that possess behavioural capabilities.

### 1.2 Contribution

This paper aims to answer a number of questions concerning the inverted pendulum, such as, how it works, how to implement it, and how it can represent a character's motion. The inverted pendulum can provide a low-dimensional approximation of a character's base movements, for example, balancing, intelligent foot placement, reacting to disturbances (i.e., pushes), and foot support transitions. To summarize the main sections of this paper:

- Independence (i.e., self-contained system)
- Limitations (i.e., pitfalls and problems)
- Step-by-Step introduction to the workings of the inverted pendulum model
- Flavors (e.g., SLIP, IP, IIP)
- Upright biped balanced stepping

- Outline limitations
- Generating fundamental motions, e.g., running, walking.
- Practical considerations (engineering enhancements)
- 2D and 3D
- Speed considerations
- Robustness and Reliability
- Ankle-Foot Region for Control
- Real-world application, i.e., bipeds, dogs, aliens (e.g., cat and centipede connecting body parts by means of springs)

## 2 Related Work

The inverted pendulum has been studied across numerous disciplines (e.g., robotics, graphics and biomechanics). Hence, to help illustrate the evolution and adoption of the inverted pendulum model for character-based solutions we present a comprehensive overview over the past few decades (see Table A). The inverted pendulum was originally a biomechanically inspired approach [Vuk72; Hem77] that later gained recognition in robotics [MS84; KT95] and later the graphics community [KKK\*01; KDM11]. Table A shows a detailed comparison grid summary of related research research involving the inverted pendulum and its adaptations across the various fields over the past few decades.

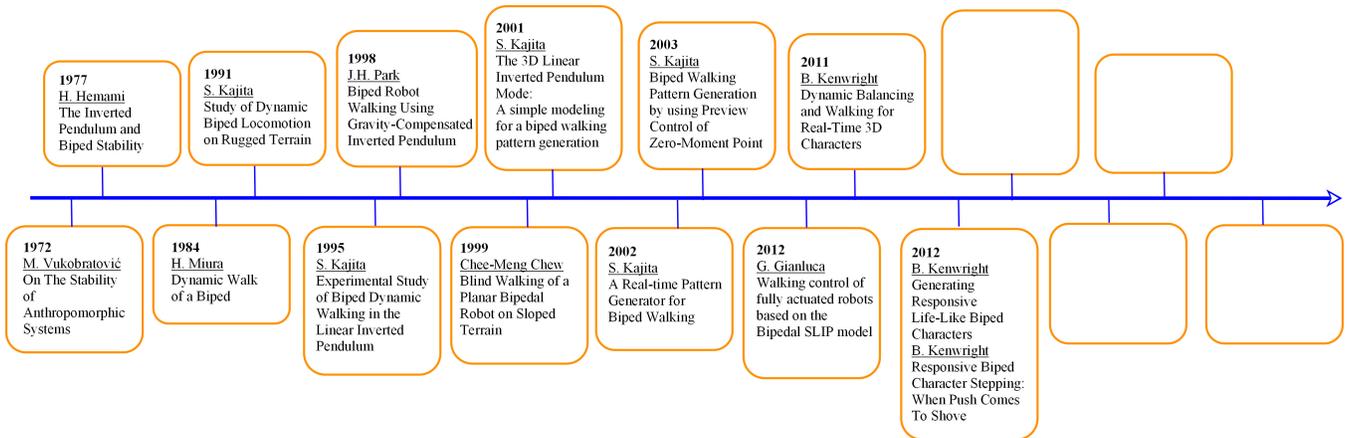
## 3 Flavors

The principle workings of the inverted pendulum (IP) is the backbone for a number of different inverted pendulum sub models, such as:

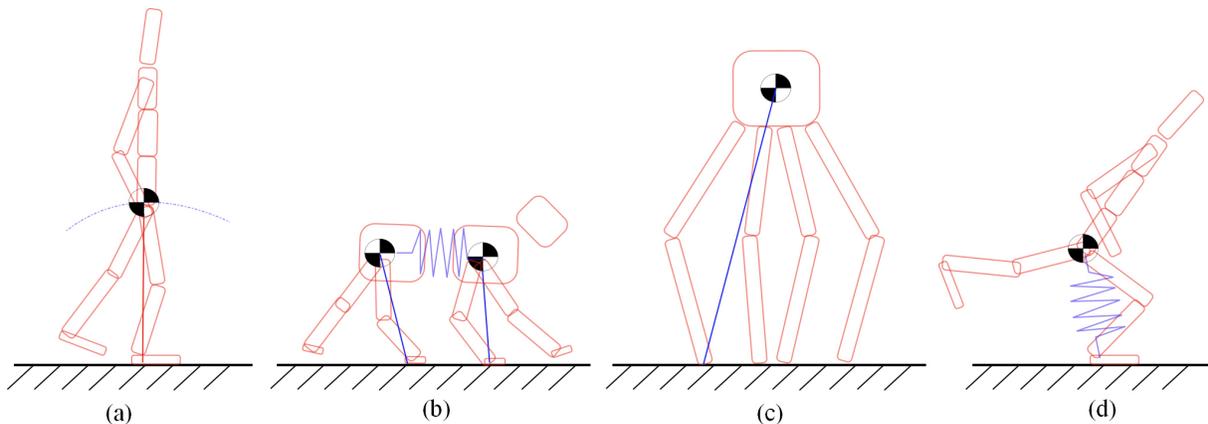
- Linear Inverted Pendulum Model (LIPM)
- Spring-Loaded Inverted Pendulum Model (SLIPM)
- Gravity-Compensated Inverted Pendulum Model (GCIPM)
- Impulse Inverted Pendulum Model (IIPM)
- Extended Linear Inverted Pendulum Model (Extended LIPM) (Elongated Rigid Body)

A number of critical simplification, assumptions, and limitations of the inverted pendulum:

- Walking is essentially a passive action
- No foot slipping or friction
- Local or World space constraints - for world space constraints the positions and orientations are corrected relative to the world while local constraints are only concerned with the local orientation and position relative to the parent
- Support region (pin-point) above COM, COP
- PD Controller for a full physics model - joint torques
- The physical properties of a human (weight, height, strength)
- Compare simulated with real-world stepping cycles from patients to identify possible walking (e.g., health) problems.
- Periodic motions
- Trajectory Transition



**Figure 3: Time-Line** - Research time-line focused on the inverted pendulum in terms of character-based solutions (e.g., biped balancing) over the past few decades.



**Figure 4: Creative adaptation of the IP model for creating balancing characters solutions** - Extending and adapting the inverted pendulum technique to different skeleton character configurations. (a) incompressible leg, (b) combined inverse pendulums (i.e., quadruped), (c) multiple legs and, (d) spring-loaded inverted pendulum (SLIP).

The pendulum movement is popular in animation since it teaches us about the wave action which happens in any multi-jointed or organic object.

## 4 Principle

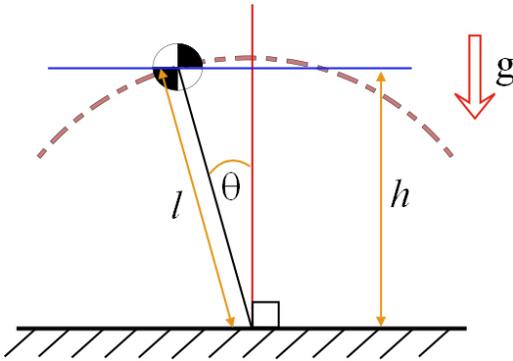
Ideally, as an inverted pendulum transitions from leg to leg and the point mass pole vaults over the foot we know that due to the mechanical law of conservation no energy will be lost as the energy is converted back and forth between kinetic and potential energy. Hence, the total mechanical energy of the inverted pendulum can be defined by Equation 4 1.

$$\begin{aligned} E_{ME} &= \text{constant} \\ &= E_{PE} + E_{KE} \\ &= mgh + \frac{1}{2}I\omega^2 \end{aligned} \quad (1)$$

where  $E_{ME}$  is the mechanical energy,  $E_{PE}$  is the potential energy,  $E_{KE}$  is the kinetic energy,  $g$  is gravity,  $h$  is height above the ground,  $m$  is the mass,  $I$  is inertia constant and  $\omega$  is the angular velocity.

We can use trigonometric functions and the geometric considerations of the IP (e.g., weightless legs and a point mass) to rewrite Equation 4 1 for the 2D case shown in Equation 4 2 (also illustrated in Figure 4 1).

$$\begin{aligned} E_{PE} &= mgl \cos(\theta) \\ I &= ml^2 \\ E_{KE} &= \frac{1}{2}ml^2\omega^2 \\ E_{ME} &= E_{PE} + E_{KE} \\ &= mgl \cos(\theta) + \frac{1}{2}ml^2\omega^2 \end{aligned} \quad (2)$$



**Figure 5: Inverted Pendulum Pogo-Stick Principle.** - The IP model is analogous to the workings of a pogo-stick and pole-vaulting mechanism.

Walking - foot must always be in contact with the ground. The IP model is a point mass so does not possess any angular momentum.

**State machine logic** A common mechanism for controlling the IP model is a state-machine logic.

Interpolation - Bezier spline and transition times

## 5 Do we need Feet?

While the IP model is a good model for emulating locomotion in different character based models (e.g., bipeds) the addition of feet can make the model more robust and provide additional control parameters. The feet do not necessarily have to be perfect representations of real feet. Instead, similar to the inverted pendulum making numerous approximations to form a usable model we can apply the same technique to add feet control to our IP model. For example, we can add a force feedback from the foot when it is in contact with the ground. We can extend the feet contact point from a single point to a circular region to provide a larger support region and produce a less jittery (i.e., constantly taking small steps for tiny disturbances) result.

## 6 Engineering Enhancements

The complete and viable implementation of the IP for a character based system can be challenging and complex since it has to incorporate additional features such as foot avoidance, trajectory stepping information, corrective feedback, and numerical inaccuracies.

### 6.1 Foot Placement Constraints - Crossed Legs, Shortest Trajectory, Orientation, Comfort

While we can determine a foot placement position that will result in the character remaining upright and balanced during standing (or locomotion), we also need to determine a trajectory that is natural, feasible, life-like for the foot placement transition. For example, we need to avoid collisions with the body (such as the other legs) and follow a trajectory that takes a comfortable path and will not result in our character having crossed legs. A typical naive implementation would use an uncomplicated 'as the crow flies' shortest path assumption based on a current direction desire (e.g., following a simple path). 7.1.1. Step-by-Step Approach Humans have spaghetti legs; they can twist, bend, and move in a vast number of ways that makes it difficult to identify desirable, comfortable, realistic looking results. However, if we take a simplified view on the problem and introduce a number of limiting constraints we can formulate an initial approximation from which we can build upon.

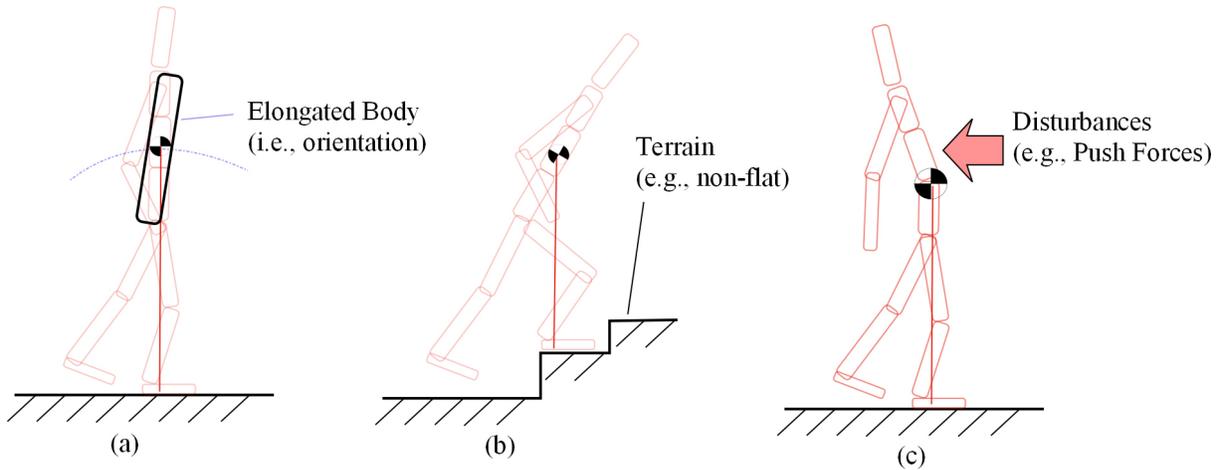
### 6.2 Examples - Hairy Dwarf

Dwarf skeleton with a simple IK solver - then upright balancing motions that can handle disturbances and a variety of terrain (e.g., pushes and non-flat ground). Skinning with fur and some random motions to make the model life-like and more interesting then combined with the IP model to produce a controllable, dynamic, interactive, flexible character.

## 7 Implementation (Step-by-Step)

We give a beginners' step-by-step introduction to understanding how the inverted pendulum works and how it can be used to generate intelligent, dynamic, and responsive stepping motions for virtual characters. We focus on the spring loaded inverted pendulum (SLIP) since this is one of the simplest to implement with excellent results. We start at the very beginning and introduce the basic mechanical reasoning and workings for the spring loaded inverted pendulum mode.

- Step 1 - Laws of Movement
- Step 2 - Hooke's Law
- Step 3 - Falling Over



**Figure 6: Beyond the basics** - We go beyond a basic uncomplicated inverted pendulum situations and show more diverse and interactive cases that demonstrate the dynamic potential of a physics-based model.

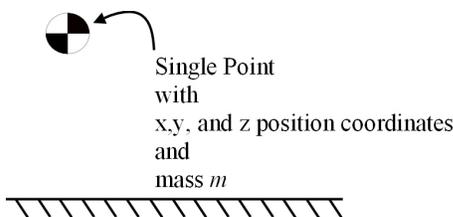
- Step 4 - Swapping Legs (Faster or Slower Locomotion)

**Step 1 - Laws of Movement** We start with the basics. Since the inverted pendulum is a physics-based model, we primarily use Newton's laws of motion. As shown in Equation 3, we can relate how fast an object accelerates and hence moves over time in relation to how much force is applied to it.

$$f = ma \quad (3)$$

where  $f$  is the force,  $m$  the mass and  $a$  is acceleration. Note that both force and acceleration are vectors while the mass is a scalar quantity.

In 3D, we can represent a particle mass by its position and mass as shown in Figure 7. Exploiting Newton's laws we can apply forces to the point mass and calculate how much acceleration and hence how much movement will be produced.



**Figure 7: Particle Point Mass** - Simplification of a complex articulated body to a single point mass with only translational information.

By means of simple integration we can predict where the particle will as time progresses due to forces acting on the body (see Figure 8).

Euler's first order integration is given in Equation 4.

$$\begin{aligned}
 f &= ma \quad (\text{from Newton's laws}) \\
 a &= \frac{1}{m} f \\
 a(t+1) &= \frac{1}{m} f \\
 v(t+1) &= v(t) + a(t+1)dt \\
 p(t+1) &= p(t) + v(t+1)dt
 \end{aligned} \quad (4)$$

where  $a$ ,  $v$ , and  $p$ , represent the acceleration, velocity and position while the  $(t)$  and  $(t+1)$  indicate the current and next time, and  $dt$  is the time step that represents the elapsed time since the previous update (typically 1/60th of a second for real-time applications).

Essentially, we have a free moving particle object. We can apply directional forces in 3D and have the object move realistically. Gravity is a directional force pointing down the  $y$ -axis. Furthermore, we can add contact and collision forces from environmental interactions.

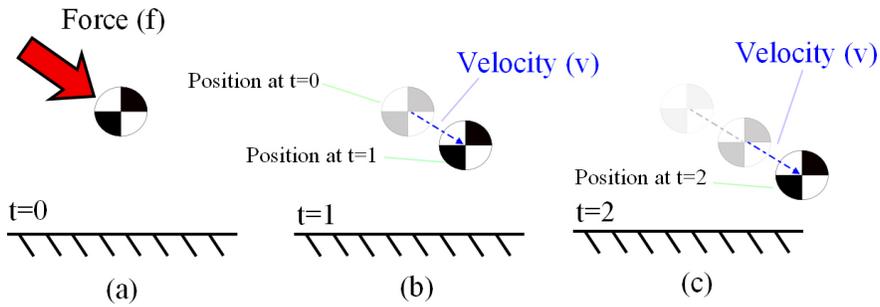
**Step 2 - Hooke's Law** We need to incorporate a quantity of constraints. These constraints are done through forces generated from spring constraints. Since the inverted pendulum is essentially a point-mass sitting on a mass-less stiff leg, we need to extend the point mass from the previous step to incorporate it.

$$f = (l_{rest} - l_{current})k_s \quad (5)$$

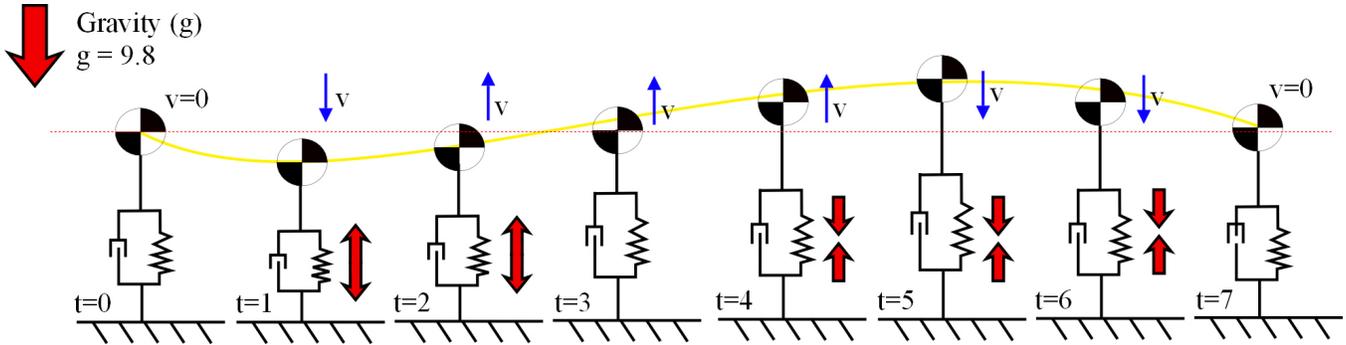
where  $f$  is the spring force,  $l_{rest}$  is the rest length and the current length of the spring,  $k_s$  is the spring stiffness constant.

We include damping into the spring so that it will not continue to oscillate and we can create a more flexible and stable inverted pendulum model. If we constantly apply gravity to the mass with the spring-damper as shown in Figure 9, the point mass will oscillate up and down as shown in Figure 10. However, with a damping constant applied to the spring it will eventually settle down and converge on a stable position.

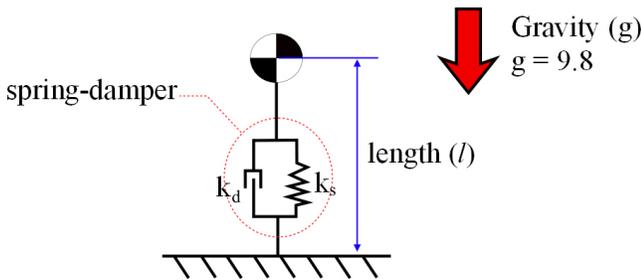
The point-mass suspended by the spring-damper is the fundamental working component of the spring-loaded inverted pendulum model.



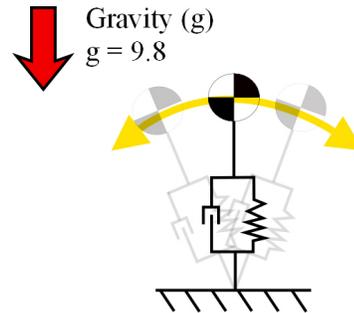
**Figure 8: Free moving particle mass.** - The particle mass follows the strict mechanical laws (i.e., forces, momentum, gravity).



**Figure 10: Oscillatory Point Mass** - The oscillatory nature of the spring-damped mass will cause it to bouncing up and down.



**Figure 9: Spring-Damper Setup** - Extend the uncomplicated raw and simple particle mass and attach a unsophisticated spring-damper mechanism.



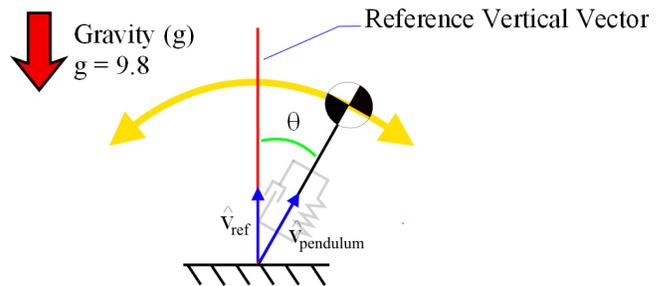
**Figure 11: Gradually Falling Over** - The balancing point-mass leg will eventually begin to rotate to the left or right (i.e., fall over).

**Step 3 - Falling Over** A point mass suspended above a spring-damper will eventually, due to numerical approximation errors, start to move towards the side (see Figure 11).

Without any intervention our spring-loaded inverted pendulum will just fall over. As the spring-leg moves towards either side the angular difference between the current or reference upward vector can be measured. The spring-leg angle and point-mass velocity are used to determine where we should reposition the spring-leg. We assume a weightless leg that can be moved instantly. Whereby, each frame we examine the models status and decide what action to take. If necessary calculate the new angle and reposition the spring-leg to achieve a specific action (i.e., walking, running, stopping).

We can use the dot product as a means of calculating the angular difference between the current and reference vector shown in Equation 6.

$$\cos(\theta) = \text{Dot}(\hat{v}_{ref}, \hat{v}_{pendulum}) \quad (6)$$



**Figure 12: Gravity Vector** - The vertical vector is the direction of gravity and the reference direction from which we calculate the angular offset.

**Step 4 - Swapping Legs (Faster or Slower Locomotion)** We can prevent our spring-loaded inverted pendulum mass from falling over by instantly moving the leg to a new position when the angle passes a specific threshold (see Figure 14). As the point-mass falls it gathers momentum. When the angle reaches or surpasses the threshold we instantly flip the leg. The point-mass continues to pole-vault forward and upwards. We check if the angle is equal or greater due to numerical errors and discrete approximations between time step updates.

Depending on if the repositioned spring-leg has an angle greater or less than the previous reference angle affects if the motion of the point-mass will slow down or speed up. For example, if the angle of the repositioned spring-leg is greater it will have insufficient momentum to pole vault over the summit peak.

## 8 Results

The simulation results can be based on real-world values. Whereby, the mass can be chosen to be that of an average human (e.g., 60kg's). Furthermore, the leg stiffness can be adjusted based on how much you would like the character's knee to bend or change. For example, a character travelling at a larger velocity (i.e., running) will have his legs bend a great deal more than if he is just standing around taking very little steps.

The 'basic' implementation presented will iteratively take small steps to remain upright and balanced. Depending upon the angle threshold and the swapped angle determines what motion the resulting stepping motion (e.g., walking, stopping). The inverted pendulum model is flexible enough to handle dynamic and uneven terrain [CPP99] (e.g., walking across a bridge or non-flat ground).

The Euler integration method is minimalistic and computationally fast; however, it can be unstable and add energy to the system. This is remedied by ensuring a sufficiently small time-step is used and that damping is added to ensure oscillations die away. In a perfect system with no damping or numerical errors the locomotive properties of the inverted pendulum model can ensure a persistent steady walk. Since we do not have a perfect system, we must constantly analyse the situation (e.g., velocity, momentum, angle) and take corrective measures each frame to ensure a constant, controllable and robust system.

## 9 Conclusion and Discussion

The inverted pendulum is a tool like any other that can be incorporated into a character-based system to produce more dynamic, interactive, and intelligent motions. While this paper has primarily focuses on generating basic upright motions the inverted pendulum can also be an external tool to provide information for animators to enable them to identify if motions are incorrect.

The fundamental spring loaded inverted pendulum is stable and flexible enough to be incorporated into any virtual environment due to its simplicity and computationally minimalism. The model can also be extended with additional bells and whistles to achieve a more varied type of motions. For example, different leg sizes to emulate a character limping, or combined with random coherent noise to incorporate more realistic and life-like motions instead of being robot like.

## 10 Further Work

This paper has focused on presenting a step-by-step no-nonsense explanation of the inverted pendulum for use in character-based

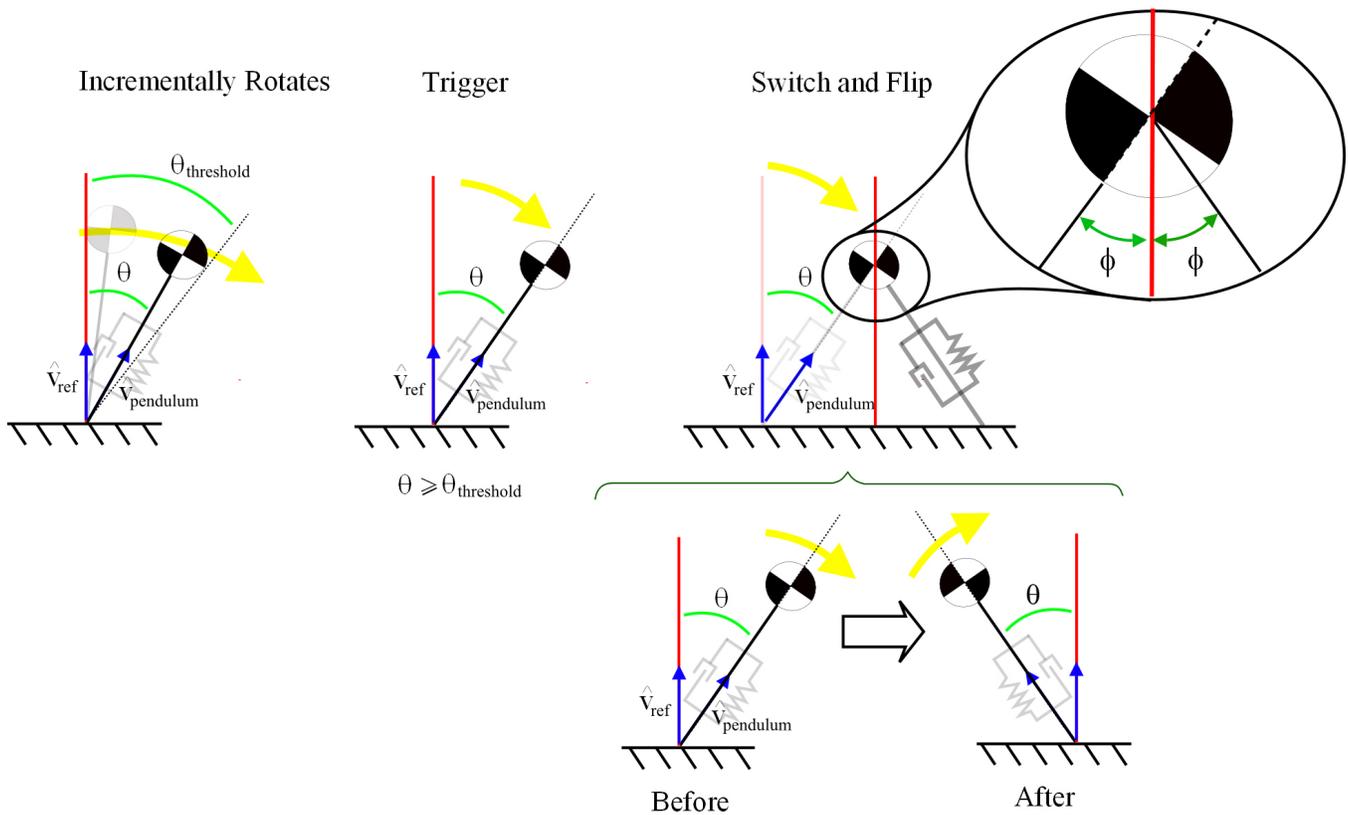
systems either as a way of providing corrective information for correcting motion capture data or as an alternative for generating the motions purely on the fly. The paper has only scratched the surface of dynamic interactive characters and future work should be on the coupling of the low-dimensional inverted pendulum model with the highly non-linear articulated skeleton through inverse kinematics and rigid body torque feedback systems (e.g., proportional derivative controller).

## 11 Acknowledgments

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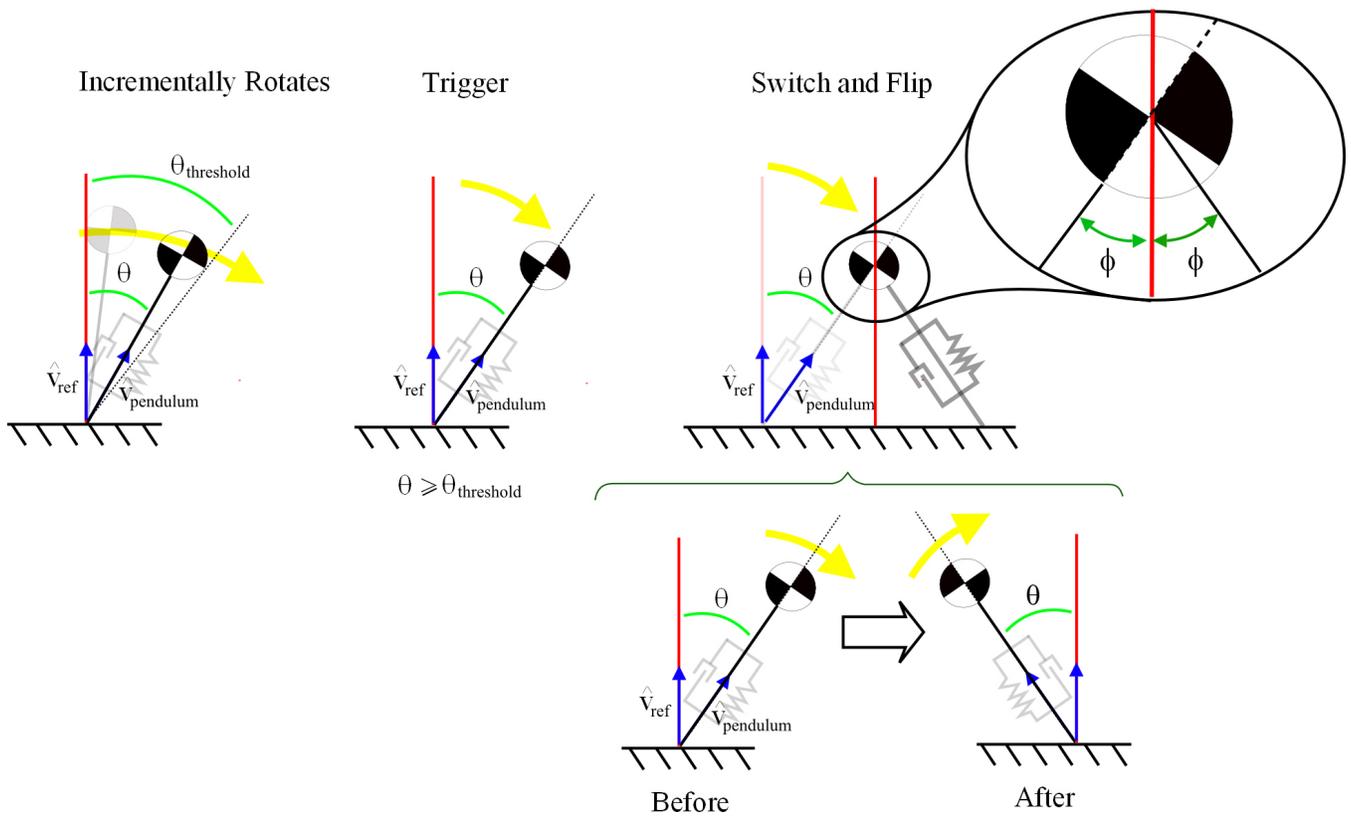
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**Figure 13:** Transition phase as the point-mass spring-leg rotates and touches or surpasses the threshold the leg is 'instantly' moved.

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**Figure 14: Stepping Logic** - Transition phase as the point-mass spring-leg rotates and touches or surpasses the threshold the leg is 'instantly' moved.

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## A Appendix

A few papers that have made a big impact in the research community and should be given credit for extending and developing crucial principles and ideas that are the backbone of many of the later papers:

	Year	Details
[Vuk72]	1972	IP B 3D Independent Roll-Pitch (no Yaw)
[Hem77]	1977	IP B 2D
[MS84]	1984	IP R 3D Independent Roll-Pitch (no Yaw)
[KT91]	1991	LIPM R 2D
[KT95]	1995	LIPM R 2D
[PK98]	1998	GCIPM R 2D
[CPP99]	1999	LIPM R 2D Sloped Terrain
[KKK*01]	2001	3D-LIPM G 3D
[KKK*02]	2002	3D-LIPM R 3D Generating Walking Pattern
[KKK*03]	2003	3D-LIPM+ZMP R 3D
[KDM11]	2011	SLIPM G 3D Procedural Motion with Postural Feedback Control
[GOAS12]	2012	SLIPM R 3D
[KGMB12]	2012	LIPM G 3D Procedural Locomotion
[Ken12a]	2012	SLIP G 3D Hybrid Life-Like Motions through Feedback Noise

Paper	Year	Title	Real-Time	Dims	Field	Cite-Count	IP-Model	Terrain	Details
[Vuk72]	1972	On The Stability of Anthropomorphic Systems	?	3D	B	?	IP	?	Independent Roll-Pitch (no Yaw)
[CFLP77]	1977	Early Recognition of Postural Disorders in Multiple Sclerosis Through Movement Analysis: A Modeling Study	?	?	?	?	?	?	?
[Hem77]	1977	The Inverted Pendulum and Biped Stability	?	2D	B	?	IP	?	?
[MS84]	1984	Dynamic Walk of a Biped	?	3D	R	?	IP	?	Independent Roll-Pitch (no Yaw)
[BC89]	1989	Goal-Directed, Dynamic Animation of Human Walking	?	?	?	?	?	?	?
[KT91]	1991	Study of Dynamic Biped Locomotion on Rugged Terrain: Derivation and Application of the Linear Inverted Pendulum Mode	?	2D	R	?	LIPM	?	
[KT95]	1995	Experimental Study of Biped Dynamic Walking in the Linear Inverted Pendulum Mode	?	2D	R	?	IP	?	
[PK98]	1998	Biped Robot Walking Using Gravity-Compensated Inverted Pendulum Mode and Computed Torque Control	?	2D	R	?	CGCIPM	?	
[MVJC99]	1999	Motion Levels of Detail for Real-Time Virtual Worlds	?	?	?	?	?	?	?
[CPP99]	1999	Blind Walking of a Planar Bipedal Robot on Sloped Terrain	?	2D	R	?	LIPM	?	Sloped Terrain
[TSK]	2000	Motion Balance Filtering	?	?	?	?	?	?	?
[KKK*01]	2001	The 3D linear inverted pendulum mode: a simple modeling for a biped walking pattern generation	✓	3D	G	323	3D-LIPM	flat	
[HOHA01]	2001	Yet Another Humanoid Walking: Passive Dynamic Walking with Torso under Simple Control	?	?	?	?	?	?	?

[FvdP01]	2001	The virtual stuntman: dynamic characters with a repertoire of autonomous motor skills	?	?	?	?	?	?	?
[KKK*02]	2002	A Real-time Pattern Generator for Biped Walking	?	3D	R	?	3D-LIPM	?	Generating Walking Pattern
[KKK*03]	2003	Biped Walking Pattern Generation by using Preview Control of Zero-Moment Point	?	3D	R	?	3D-LIPM+ZMP	?	
[Kok]	2004	Dynamics of Human Walking at Steady Speeds	?	?	?	?	?	?	?
[KLL04]	2004	Animating Reactive Motions for Biped Locomotion	?	?	?	?	?	?	?
[YPVDP05]	2005	Data-driven interactive balancing behaviors	?	?	?	?	?	?	?
[KDR05]	2005	Energetic Consequences of Walking Like an Inverted Pendulum: Step-to-Step Transitions	?	?	?	?	?	?	?
[AFO05]	2005	Pushing people around	?	?	?	?	?	?	?
[GSB06]	2006	Compliant leg behaviour explains basic dynamics of walking and running	?	?	?	?	?	?	?
[PES06]	2006	Ballet balance strategies	?	?	?	?	?	?	?
[KKI06]	2006	Stepping Motion for a Human-like Character to Maintain Balance against Large Perturbations	?	?	?	?	?	?	?
[SKL07]	2007	Simulating biped behaviors from human motion data	?	?	?	?	?	?	?
[LCCL07]	2007	Realistic Character Motion Response in Computer Fighting Game	?	?	?	?	?	?	?
[UWW07]	2007	Mechanics of dog walking compared with a passive, stiff-limbed, 4-bar linkage model, and their collisional implications	?	?	?	?	?	?	?
[JWM07]	2007	Dynamic Humanoid Balance through Inertial Control	?	?	?	?	?	?	?
[WMZ]	2008	Simple Steps for Simply Stepping	?	?	?	?	?	?	?
[P.08]	2008	Momentum-Based Balance Control For Simulated Characters	?	?	?	?	?	?	?
[MZO9]	2009	Momentum Control for Balance	?	?	?	?	?	?	?
[MAA*]	2009	Passive Dynamic Walking with Torso under Simple Control	?	?	?	?	?	?	?
[WA09]	2009	Control of a Walking Biped Using a Combination of Simple Policies	?	?	?	?	?	?	?
[LA]	2009	Standing balance control using a trajectory library	?	?	?	?	?	?	?
[XCJ*09]	2009	Human Walking Capture and its Inspiration to Biped Gait Planning	?	?	?	?	?	?	?
[VBPE]	2010	The step space: example-based footprint-driven motion synthesis	?	?	?	?	?	?	?
[SKST10]	2010	Physically-Based Character Control in Low Dimensional Space	?	?	?	?	?	?	?
[KH10]	2010	Control Systems for Human Running using an Inverted Pendulum Model and a Reference Motion Capture Sequence	?	?	?	?	?	?	?
[CBVDP10]	2010	Generalized Biped Walking Control	?	?	?	?	?	?	?
[WZ10]	2010	Goal-directed stepping with momentum control	?	?	?	?	?	?	?
[MDLH10]	2010	Robust physics-based locomotion using low-dimensional planning	?	?	?	?	?	?	?
[TLC*10]	2010	Real-Time Physics-Based 3D Biped Character Animation Using an Inverted Pendulum Model	?	?	?	?	?	?	?
[KDM11]	2011	Dynamic Balancing and Walking for Real-Time 3D Characters	✓	3D	G	?	SLIPM	?	Procedural Motion with Postural Feedback Control
[CHY*11]	2011	Biped Walking Planning Using Extended Linear Inverted Pendulum Mode with a Continuous Moving ZMP	?	?	?	?	?	?	?
[SKF11]	2011	Footstep navigation for dynamic crowds	?	?	?	?	?	?	?
[Ma]	2011	Generating Real-time Responsive Balance Recovery Animation	?	?	?	?	?	?	?
[Ken12a]	2012	A Beginners Guide to Dual-Quaternions: What They Are, How They Work, and How to Use Them for 3D Character Hierarchies	✓	?	?	?	?	?	?
[GOAS12]	2012	Walking control of fully actuated robots based on the Bipedal SLIP model	?	3D	R	?	SLIPM	?	
[KGBM12]	2012	Procedural locomotion of multilegged characters in dynamic environments	?	3D	G	?	LIPM	?	Procedural Locomotion
[Ken12b]	2012	Generating Responsive Life-Like Biped Characters	?	3D	G	?	SLIP	?	Random Coherent Noise to Create Hybrid Life-Like Motions

**Table 1:** Comparison grid of the inverted pendulum over the years and the numerous enhancements and modifications by different research fields relating to characters and balancing motions.